$c = -2$ conformal field theory in quadratic band touching

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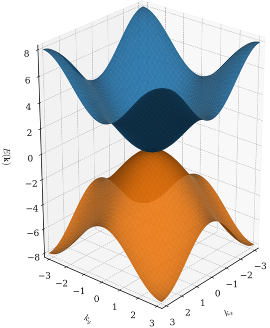
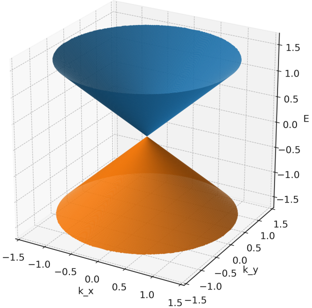
QBT defines a **distinct universality class** ($\leftrightarrow$ Dirac point).



vs



$\varepsilon \propto \pm|\boldsymbol{k}|^2$ (non-relativistic)

$\varepsilon \propto \pm|\boldsymbol{k}|$ (relativistic)

QBT appears in kagome, checkerboard, bilayer graphene, …

[Liu, Yao, Ma PRB 2010; Sun PRB 2008; McCann & Fal'ko PRL 2006; …]

What is the characterization of QBT as a quantum critical point?

Spatial Conformal Invariance at QBT

QBT is **marginally unstable** against interactions.

- Various interaction-induced phases have been predicted

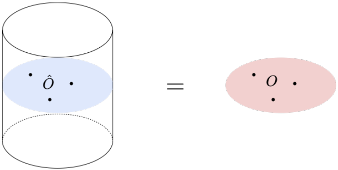
  Nematic order, quantum anomalous Hall, quantum spin Hall, …

  [Sun et al. PRL 2009; Murray & Vafek PRB 2014; Zhu et al. PRL 2016; …]

What about the non-interacting case? Nothing new, seemingly.

This work: QBT has spatial conformal invariance

cf. conformal quantum critical point [Ardonne, Fendley, Fradkin 2004]



Ground state correlators = classical field theory correlators

Continuum Model

Let us focus on 2+1D. Fermions are represented as **1-forms**:

$$\hat{\psi}(\boldsymbol{x}) = \hat{\psi}_1(\boldsymbol{x})dx^1 + \hat{\psi}_2(\boldsymbol{x})dx^2$$

Nontrivial rotationally symmetric QBT Hamiltonian is given by [Fradkin (2013)]

$$\hat{H} = t_+ \int d\hat{\psi}^\dagger \wedge \star d\hat{\psi} + t_- \int \delta\hat{\psi} \wedge \star\delta\hat{\psi}^\dagger$$

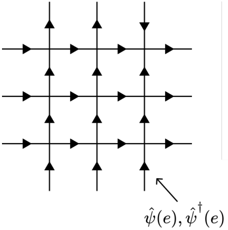
without chemical potential term (half-filling).

- $d\hat{\psi}$: exterior derivative (curl)
- $\delta\hat{\psi} = -\star d \star \hat{\psi}$: codifferential (divergence)

In momentum space:

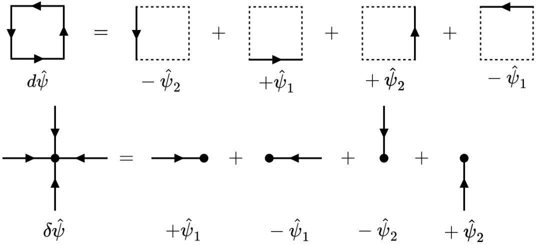
$$H(\boldsymbol{k}) = \begin{pmatrix} t_+ k_2^2 - t_- k_1^2 & (t_+ + t_-)k_1 k_2 \\ (t_+ + t_-)k_1 k_2 & t_+ k_1^2 - t_- k_2^2 \end{pmatrix}$$

Square Lattice Realization



1-form fermions

on edges



$$\hat{H} = t_+ \sum_{f \in \text{Faces}} d\hat{\psi}^\dagger(f) d\hat{\psi}(f) + t_- \sum_{v \in \text{Vertices}} \delta\hat{\psi}(v) \delta\hat{\psi}^\dagger(v)$$

Path Integral Representation of Ground State

A ground state has a **path integral representation** in terms of a Grassmann scalar $\theta(\boldsymbol{x})$:

$$|\mathrm{GS}\rangle = \frac{1}{\sqrt{Z}} \int \theta_{\boldsymbol{k}=0} |\kappa d\theta\rangle \mathcal{D}\theta$$

- Z: normalization
- κ: const ($= 1/\sqrt{4\pi}$ by convention)
- $|\kappa d\theta\rangle := e^{-\int \kappa d\theta \wedge \star \hat{\psi}^\dagger} |0\rangle$ (fermionic coherent state)

Main Result

Key step: coherent state overlap produces the symplectic fermion action

$$\langle \kappa d\theta^* | \kappa d\theta \rangle = \exp\left(\kappa^2 \int d\theta^* \wedge \star d\theta \right) =: e^{-S[\theta,\theta^*]}$$

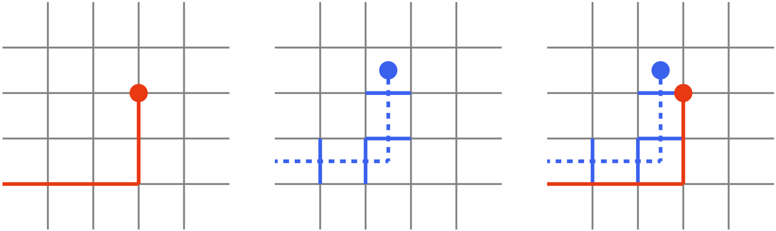
Quantum-Classical Correspondence:

$$\begin{aligned} &\langle \mathrm{GS} | F[\hat{\psi}^\dagger] G[\hat{\psi}] | \mathrm{GS} \rangle \\ &\quad = \frac{1}{Z} \int \mathcal{D}\theta \mathcal{D}\theta^* \theta^*_{\boldsymbol{k}=0} F[\kappa d\theta^*] G[\kappa d\theta] \theta_{\boldsymbol{k}=0} e^{-S[\theta,\theta^*]} \\ &\quad = \langle \theta^*_{\boldsymbol{k}=0} F[\kappa d\theta^*] G[\kappa d\theta] \theta_{\boldsymbol{k}=0} \rangle_S \end{aligned}$$

Symplectic fermion theory is CFT → Spatial conformal invariance!

2D symplectic fermion theory is a logarithmic CFT with $c = -2$

Excitations: Topological Strings



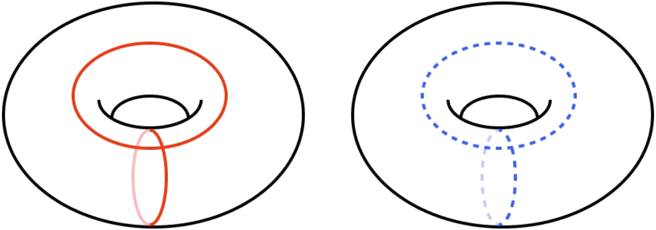
θ / electric $\quad\quad$ ϕ^* / magnetic $\quad\quad$ Composite

Excitations are created by **string operators**:

- **θ (electric)** excitation: $|\theta(\boldsymbol{x}_1)\theta(\boldsymbol{x}_2)\rangle := \kappa^{-1}\int_{\boldsymbol{x}_2}^{\boldsymbol{x}_1} \hat{\psi}|\mathrm{GS}\rangle$
- **ϕ^* (magnetic)** excitation: $|\phi^*(\tilde{\boldsymbol{x}}_1)\phi^*(\tilde{\boldsymbol{x}}_2)\rangle := \kappa^{-1}\int_{\tilde{\boldsymbol{x}}_2}^{\tilde{\boldsymbol{x}}_1} \star\hat{\psi}^\dagger|\mathrm{GS}\rangle$

Strings are **continuously deformable**.

Topological Degeneracy



Non-contractible loops on a torus

Non-contractible string operators map $|\mathrm{GS}\rangle$ to other ground states

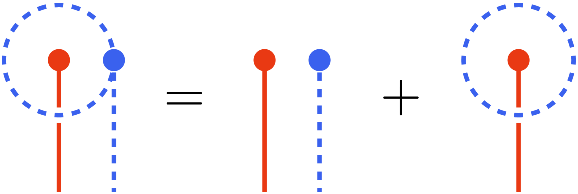
4^g**-fold ground state degeneracy**
on a genus-g surface

Analogous to the toric code, but the system is *gapless*

Non-diagonalizable Spin

Individual θ, ϕ^* excitations have **trivial spin**: they return to themselves under 2π rotation.

Rotate the composite $\phi^*\theta$ excitation by 2π:



$$R_{2\pi}(\phi^*\theta) = \phi^*\theta - 4\pi\mathbb{I}$$

$\phi^*\theta$ and $\mathbb{I}$ form a Jordan cell under rotation
the hallmark of logarithmic CFT

Summary

- ✓ QBT ground state $\Leftrightarrow$ **symplectic fermion theory**
- ✓ **Analogue of toric code**
  string operators, 4^g-fold degeneracy, smooth/rough boundaries
- ✓ Composite excitation has **non-diagonalizable spin**

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