Rigorous lower bound on dynamical exponents of critical frustration-free systems

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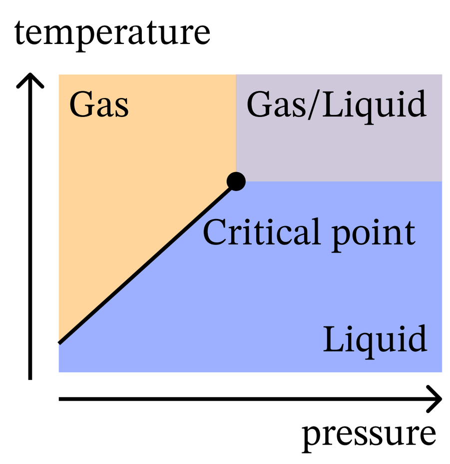
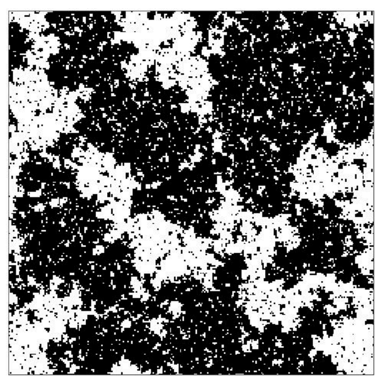
Critical phenomena

Static critical phenomena

e.g. • Critical point of H_2O • Ising model (model of magnets)

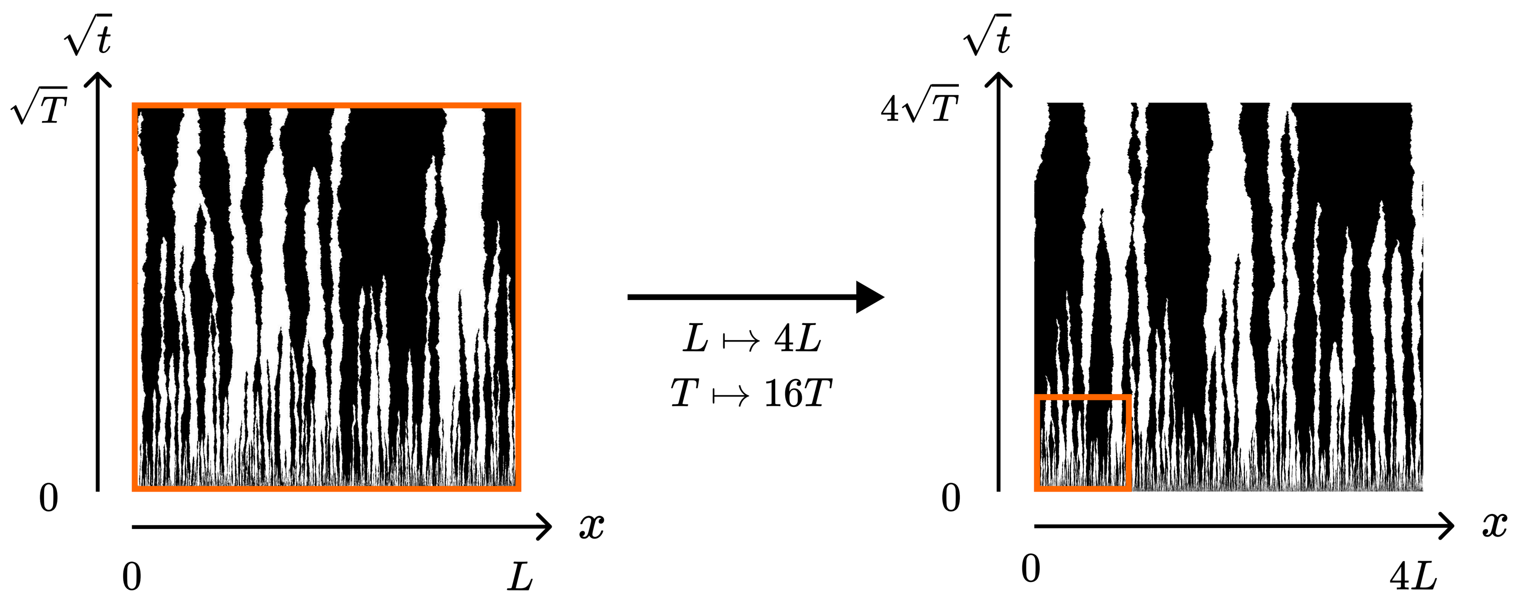




- Universality ← Liquid/gas critical point = Magnet critical point!
- Scale invariance
- Sophisticated understanding (conformal field theory)

Dynamic critical phenomena

e.g. Kinetic Ising model



- Universality
- Lifshitz scale invariance
- Strong anisotropy (time ≠ space) ← Characterized by dynamical exponent z

Def. (relaxation time) $\sim$ (system width)^z

Quantum critical phenomena

- Critical point at zero temperature due to quantum fluctuations
- Gapless
- Typical quantum critical points ≈ static critical points

Frustration-free systems

A quantum system defined by the Hamiltonian

$$\hat{H} = \sum_{i\in\Lambda} \hat{H}_i$$

is called frustration-free (FF) iff all local terms are simultaneously minimizable.
→ Solvability of ground states.

Condensed matter physics perspective
FF systems can approximate gapped quantum phases [1].

Non-equilibrium statistical physics perspective
FF systems describes standard relaxation processes with detailed balance [2].

Quantum information perspective
Determining FF-ness is the quantum analogue of k-SAT problem.

Our result

Our study focuses on FF quantum critical points.

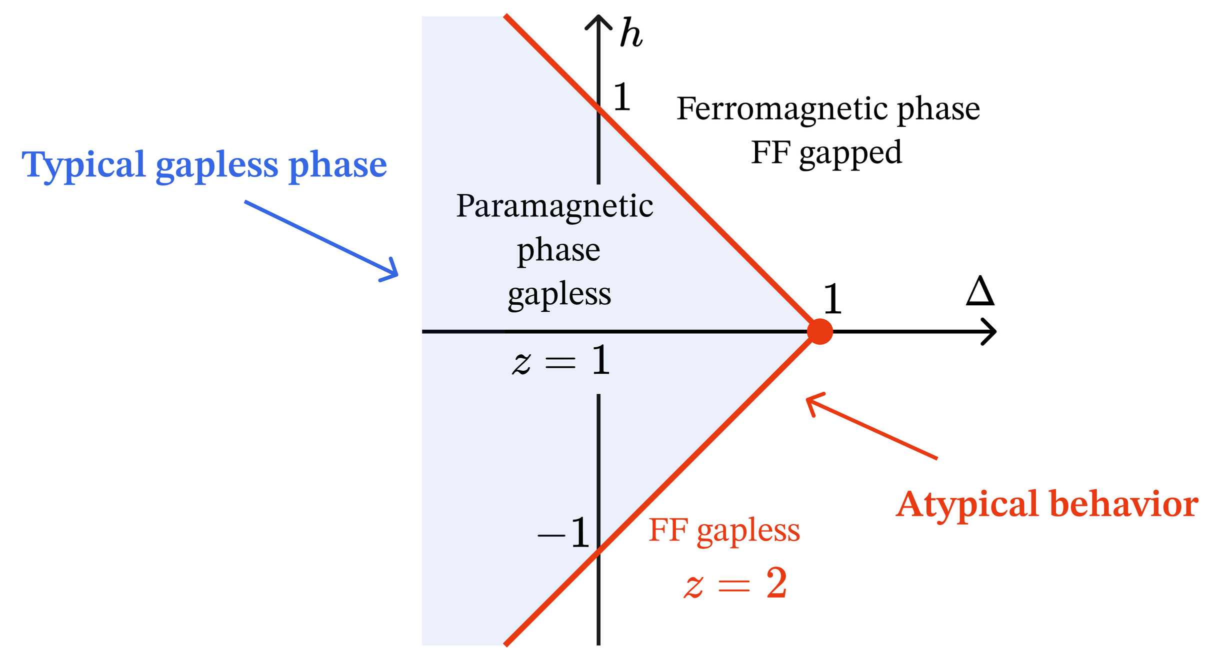
Quantum critical points are characterized by dynamical exponent z.

Def. $\epsilon =$ (energy gap) $\sim$ (system width)$^{-z}$

- Typical behavior: $z = 1$ (time ≈ space)
- FF critical points: $z \geq 2$ (time ≠ space) ← no complete proof

e.g. XXZ chain with magnetic field:

$$H = -\sum_i (X_i X_{i+1} + Y_i Y_{i+1} + \Delta Z_i Z_{i+1}) - 2h \sum_i Z_i$$

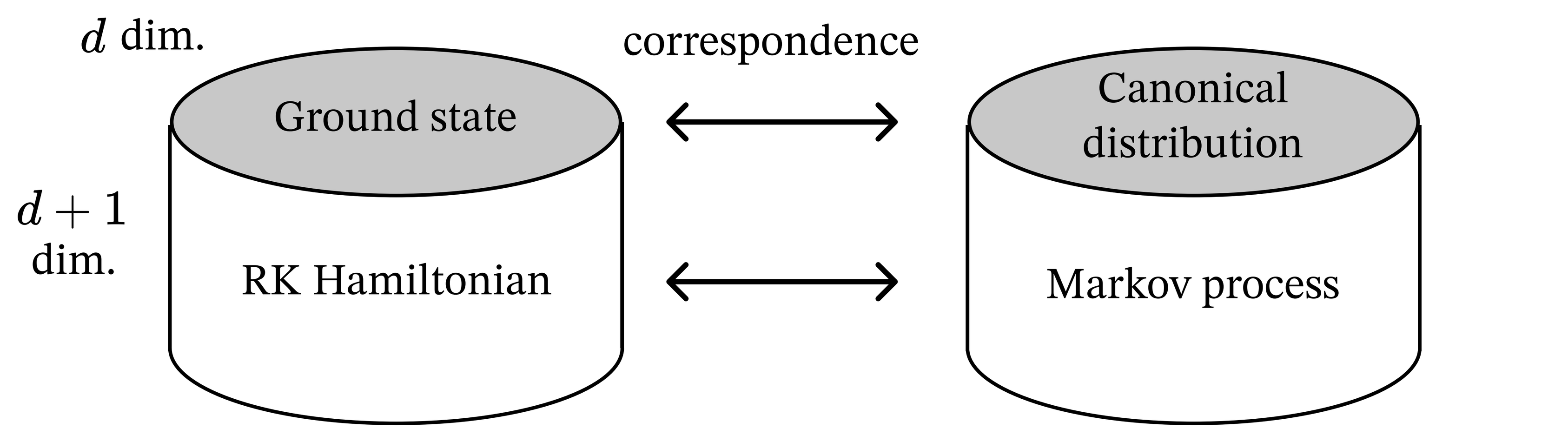


- In Ref. [3], we rigorously showed that $z \geq 2$ for a broad class of FF critical points.
- The key inequality [4]:

$$\frac{|\langle\Psi|\hat{\mathcal{O}}(\hat{\mathbb{I}} - \hat{G})\hat{\mathcal{O}}'|\Psi\rangle|}{\|\hat{\mathcal{O}}|\Psi\rangle\|\|\hat{\mathcal{O}}'|\Psi\rangle\|} \leq 2e^2 \exp\left(-\frac{D(\hat{\mathcal{O}}, \hat{\mathcal{O}}') - 1}{c-1}\sqrt{\frac{\epsilon}{\epsilon + g^2}}\right)$$

Our proof of $z \geq 2$ is also valid for various dynamic critical points.

Rokhsar-Kivelson Hamiltonians (⊂ FF) ⇔ Markov process with detailed balance [2]



Equilibrium states	Dynamical exponent z
Ising (2D)	2.1667(5)[5]
Ising (3D)	2.0245(15)[6]
Heisenberg (3D)	2.033(5)[7]
Three-state Potts (2D)	2.193(5)[8]
Four-state Potts (2D)	2.296(5)[9]

TABLE: Numerical values of z for various dynamic critical points with detailed balance

→ First proof of the empirical fact in the non-equilibrium statistical physics.

Open questions

- Proof of $z \geq 2$ for phase ordering kinetics in $d \geq 2$.
- Field theoretic justification of $z \geq 2$ for stochastic quantization [10].
- Mechanism to protect FF critical points
- Non-Hermitian cases (ASEP has FF-like property but $z = 3/2 < 2$ under PBC)

References

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