Logarithmic conformal field theory in quadratic band touching system

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Introduction

Conformal field theory (CFT) is a powerful tool for quantum critical points.
However, it assumes Lorentz invariance.

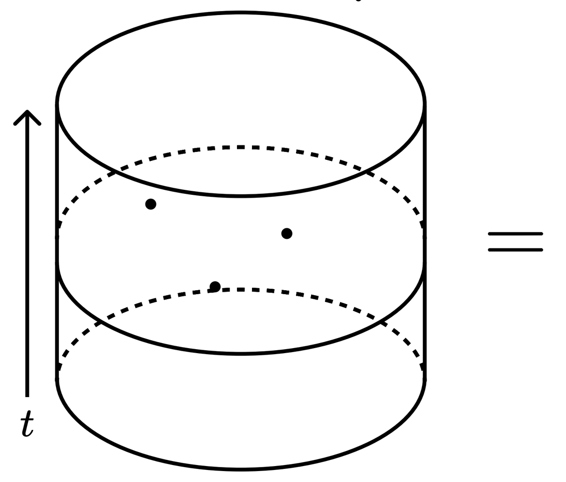
Conformal quantum critical points (CQCP) [1] in (d+1)-dimension:

- A class of quantum critical points
- No Lorentz invariance ⇒ No (d+1)-dim. conformal invariance
- Ground state (GS) exhibits d-dim. conformal invariance
- (Frustration-free) ← Not necessary, but ensures non-relativistic behavior [2]

Construction of CQCP by Rokhsar-Kivelson states [3]

Quantum-classical correspondence:

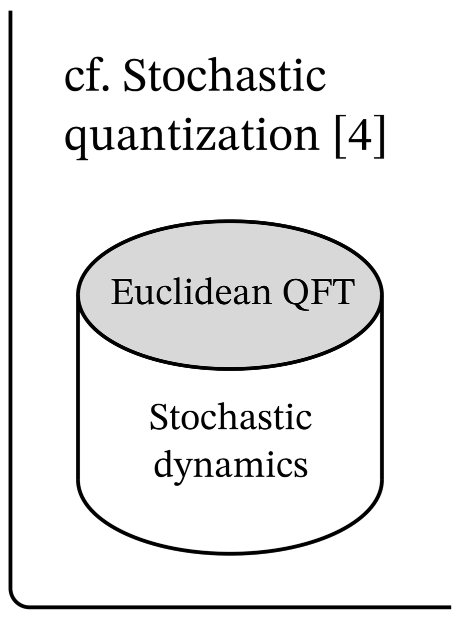
$\langle \mathrm{GS}|\hat{O}|\mathrm{GS}\rangle_{\mathrm{CQCP}}$ = $\langle O\rangle_{\mathrm{CFT}}$



(d+1)-dim. CQCP

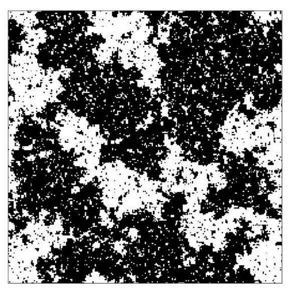
d-dim. CFT

cf. Stochastic quantization [4]

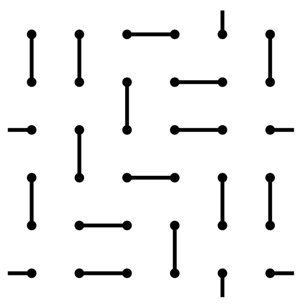


Rokhsar-Kivelson states $|\mathrm{RK}\rangle = \int \mathcal{D}\phi\, e^{-S_{\mathrm{CFT}}[\phi]/2}|\phi\rangle$ satisfy this correspondence.
→ Parent Hamiltonian of $|\mathrm{RK}\rangle$ is CQCP.

Examples



- Kinetic Ising model
  $$\hat{H} = \sum\nolimits_i \left(\hat{X}_i - e^{-J\sum_{j\sim i}\hat{Z}_i\hat{Z}_j}\right)$$
  $$|\mathrm{GS}\rangle = \sum_s \exp\left(\frac{J}{2}\sum_{\langle i,j\rangle} s_i s_j\right)|s\rangle \quad \leftrightarrow \text{2D Ising CFT}$$



- Rokhsar-Kivelson point of quantum dimer model [5]
  $$\hat{H}_i = \frac{1}{2}(|\text{⫾⫾}\rangle - |\text{⚌}\rangle)(\langle\text{⫾⫾}| - \langle\text{⚌}|)$$
  $$|\mathrm{GS}\rangle = \sum|\mathrm{Dimer}\rangle \quad \leftrightarrow \text{2D free boson CFT}$$

→ bosonic

What about **fermionic** models? (RK states do not work for Grassmann variables)

I found that a (2+1)-dim. free fermion with a singular quadratic band touching is a CQCP whose ground states correspond to the CFT known as the symplectic fermion.

Symplectic fermion

Logarithmic CFT [6]

Virasoro generators have a Jordan cell.

$$[L_k, \Phi_h(z)] = z^k[z\partial\Phi_h(z) + (k+1)h\Phi_h(z)]$$
$$[L_k, \Psi_h(z)] = z^k[z\partial\Psi_h(z) + (k+1)(h\Psi_h(z) + \Phi_h(z))]$$

Transformation of fields:

$$\Phi_h(z) \mapsto \left(\frac{df}{dz}\right)^h \Phi_h(f(z))$$
$$\Psi_h(z) \mapsto \left(\frac{df}{dz}\right)^h \left(\Psi_h(f(z)) + \log\left(\frac{df}{dz}\right)\Phi_h(f(z))\right)$$

Ψ is called the logarithmic partner of Φ

2-pt function in ordinary CFT:

$$\langle\Phi_h(z)\Phi_{h'}(w)\rangle = \delta_{hh'}\frac{A}{(z-w)^{h+h'}}$$

2-pt function in log CFT:

$$\langle\Phi_h(z)\Phi_{h'}(w)\rangle = 0$$
$$\langle\Phi_h(z)\Psi_{h'}(w)\rangle = \delta_{hh'}\frac{A}{(z-w)^{h+h'}}$$
$$\langle\Psi_h(z)\Psi_{h'}(w)\rangle = \delta_{hh'}\frac{B - 2A\log(z-w)}{(z-w)^{h+h'}}$$

Symplectic fermion [7]

$$S[\bar\theta, \theta] = -\int \mathrm{d}^2x\,\partial_i\bar\theta\,\partial^i\theta, \quad \theta, \bar\theta : \text{Grassmann-valued fields}$$

- Logarithmic partner of identity: $\tilde{\mathbb{I}} = 2\pi{:}\bar\theta\theta{:}$

$$\langle\mathbb{I}\mathbb{I}\rangle = 0, \quad \langle\mathbb{I}\tilde{\mathbb{I}}\rangle = 1, \quad \langle\tilde{\mathbb{I}}(z,\bar z)\tilde{\mathbb{I}}(w,\bar w)\rangle = -2\log|z-w|$$
$$\langle\partial\bar\theta\,\partial\theta\rangle = 0, \quad \langle\tilde{\mathbb{I}}\,\partial\bar\theta(z)\partial\theta(w)\rangle = \frac{1}{2\pi(z-w)^2}$$

- Central charge: $c = -2$ (which implies non-unitarity)

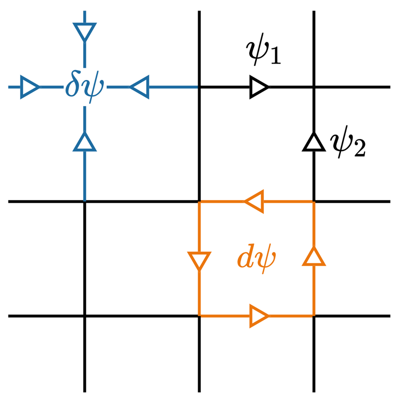
Observed in

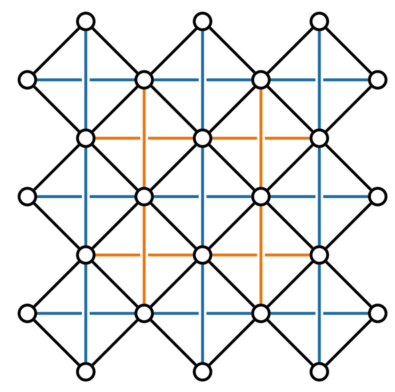
- Abelian sandpile model,
- Haldane-Rezayi state in $\nu = 5/2$ quantum Hall effect,
- Non-Hermitian Su-Schriefer-Heeger model, etc.

Model

Consider fermions on the edges of the square lattice.
Hamiltonian:

$$\hat{H} = \sum_{v\in V} d\hat\psi^\dagger(v)\,d\hat\psi(v) + \sum_{f\in F}\delta\hat\psi(f)\,\delta\hat\psi^\dagger(f)$$





Continuum model:

$$H(\boldsymbol{k}) = \begin{pmatrix} k_2^2 - k_1^2 & -2k_1k_2 \\ -2k_1k_2 & k_1^2 - k_2^2 \end{pmatrix}$$

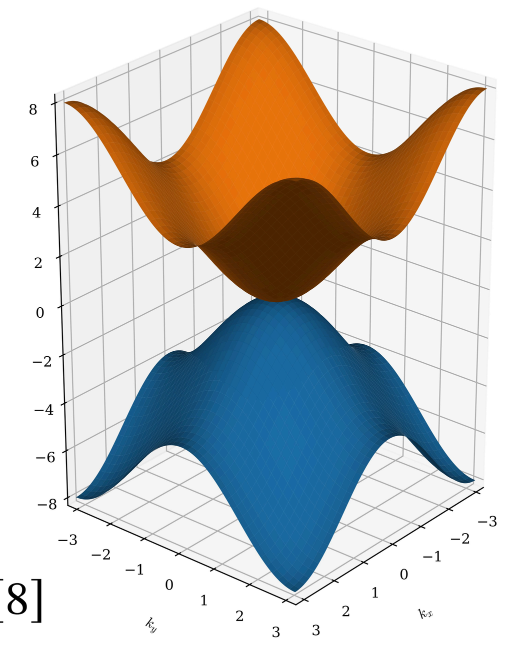
$$\hat{H} = \int (d\hat\psi^\dagger \wedge \star d\hat\psi + \delta\hat\psi \wedge \star\delta\hat\psi^\dagger),$$

$\hat\psi = \hat\psi_1 \mathrm{d}x^1 + \hat\psi_2 \mathrm{d}x^2$,
$d\hat\psi = (\partial_1\hat\psi_2 - \partial_2\hat\psi_1)\mathrm{d}x^1 \wedge \mathrm{d}x^2$
$\delta\hat\psi = -(\partial_1\hat\psi_1 + \partial_2\hat\psi_2)$

Band dispersions: $\epsilon_\pm(\boldsymbol{k}) = \pm|\boldsymbol{k}|^2$

Bloch states: $b_+(\boldsymbol{k}) = \begin{pmatrix} -k_2/|\boldsymbol{k}| \\ k_1/|\boldsymbol{k}| \end{pmatrix}$, $b_-(\boldsymbol{k}) = \begin{pmatrix} k_1/|\boldsymbol{k}| \\ k_2/|\boldsymbol{k}| \end{pmatrix}$

→ singular at $\boldsymbol{k} = \boldsymbol{0}$, $\quad d_\pm^{\max} = \lim_{k\to 0}\max_{|\boldsymbol{k}|=|\boldsymbol{k}'|=k}(1 - b_\pm(\boldsymbol{k})\cdot b_\pm(\boldsymbol{k}')) = 1$



Quadratic band touching is protected by C_4 (or C_6) symmetry [8]

Explicit correspondence

Ground state is written as (by ignoring zero modes):

$$|\mathrm{GS}\rangle = \prod\nolimits_{\boldsymbol{k}\neq 0}\frac{ik^j}{|\boldsymbol{k}|}\hat\psi^\dagger_{\boldsymbol{k},j}|0\rangle = \frac{1}{\sqrt{Z}}\int\mathcal{D}\theta\,\theta_{\boldsymbol{k}=\boldsymbol{0}}\exp\left(\int\frac{d^2\boldsymbol{k}}{(2\pi)^2}\theta_{\boldsymbol{k}}ik^j\hat\psi^\dagger_{\boldsymbol{k},j}\right)|0\rangle$$
$$= \frac{1}{\sqrt{Z}}\int\mathcal{D}\theta\,\theta_{\boldsymbol{k}=\boldsymbol{0}}\exp\left(\int d^2x\,\partial^i\theta(x)\hat\psi^\dagger_i(x)\right)|0\rangle$$
$$=: \frac{1}{\sqrt{Z}}\int\mathcal{D}\theta\,\xi|d\theta\rangle.$$

← satisfies frustration-freeness: $d\hat\psi|\mathrm{GS}\rangle = \delta\hat\psi^\dagger|\mathrm{GS}\rangle = 0$.

Norm of GS:

$$\langle\mathrm{GS}|\mathrm{GS}\rangle = \frac{1}{Z}\int\mathcal{D}\theta\mathcal{D}\bar\theta\,\bar\xi\xi\exp\left(\int \mathrm{d}^2x\,\partial_i\bar\theta\,\partial^i\theta\right)$$

← Action of symplectic fermion!

Two point correlation function is

$$\langle\mathrm{GS}|\hat\psi^\dagger_i(\boldsymbol{x})\hat\psi_j(\boldsymbol{y})|\mathrm{GS}\rangle = \frac{1}{Z}\int\mathcal{D}\theta\bar\xi\,\langle d\bar\theta|\hat\psi^\dagger_i(\boldsymbol{x})\hat\psi_j(\boldsymbol{y})\int\mathcal{D}\bar\theta\xi|d\theta\rangle$$
$$= \frac{1}{Z}\int\mathcal{D}\theta\mathcal{D}\bar\theta\,\bar\xi\xi\,\partial_i\bar\theta(\boldsymbol{x})\partial_j\theta(\boldsymbol{y})e^{-S[\bar\theta,\theta]}$$
$$= \langle\tilde{\mathbb{I}}\,\partial_i\bar\theta(\boldsymbol{x})\partial_j\theta(\boldsymbol{y})\rangle = -\frac{1}{2\pi}\partial_{x_i}\partial_{y_j}\ln|\boldsymbol{x}-\boldsymbol{y}|.$$

In general, $\langle\mathrm{GS}|F(\hat\psi^\dagger)G(\hat\psi)|\mathrm{GS}\rangle = \langle\tilde{\mathbb{I}}F(d\bar\theta)G(d\theta)\rangle$.

The above correspondence is summarized as

$$\hat\psi_i \leftrightarrow \partial_i\theta, \quad \hat\psi^\dagger_i \leftrightarrow \partial_i\bar\theta \qquad \text{(with logarithmic partner of identity)}$$

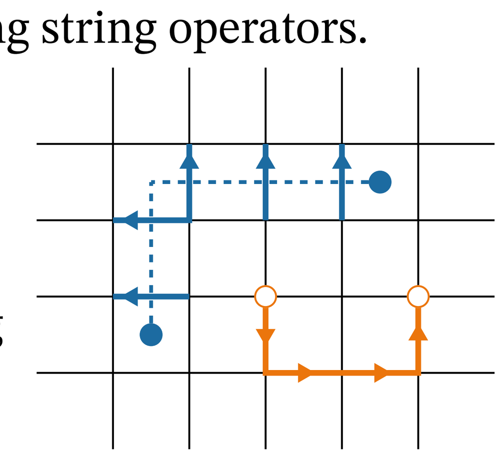
String operators

In the above correspondence, θ always appears as its derivative.
However, one can create pair of θ excitations by the following string operators.

$$|\theta(\boldsymbol{x})\theta(\boldsymbol{y})\rangle := \int\mathcal{D}\theta\,\theta(\boldsymbol{x})\theta(\boldsymbol{y})|d\theta\rangle = \int_{\boldsymbol{x}}^{\boldsymbol{y}}\hat\psi|\mathrm{GS}\rangle$$

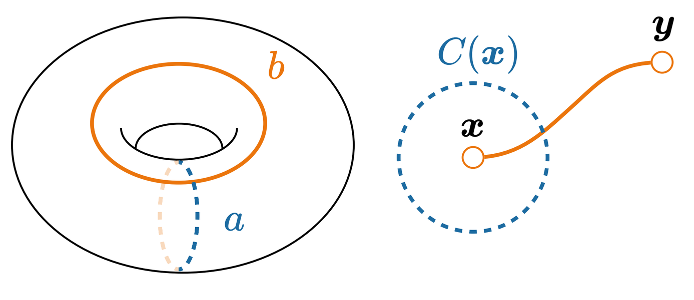
$$|\tilde\theta(\tilde{\boldsymbol{x}})\tilde\theta(\tilde{\boldsymbol{y}})\rangle = \int_{\tilde{\boldsymbol{x}}}^{\tilde{\boldsymbol{y}}}\star\hat\psi^\dagger|\mathrm{GS}\rangle$$

↑
Any path connecting two points



Moreover, string operators winding around noncontractible loops connect degenerate ground states.

$$\oint_a \star\hat\psi^\dagger|\mathrm{GS}\rangle = |\mathrm{GS}'\rangle, \quad \oint_b \hat\psi|\mathrm{GS}'\rangle = |\mathrm{GS}\rangle$$
$$\oint_{C(\boldsymbol{x})}\star\hat\psi^\dagger|\theta(\boldsymbol{x})\theta(\boldsymbol{y})\rangle = |\mathrm{GS}\rangle$$



Future directions

- Calculation of entanglement entropy using symplectic fermion (ongoing)
- Construction of other examples of fermionic CQCP (possibly with interaction)
- Introducing twist fields
- Relevant or marginal perturbations
- Do symplectic fermion GS provide any information on dynamics?

References

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