Rigorous lower bound of dynamic critical exponents in critical frustration-free systems

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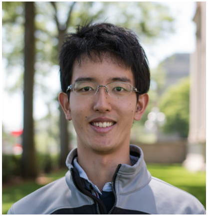
March 18, 2025

Collaborators



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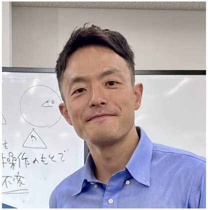

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Gapless quantum phases are characterized by the dynamic critical exponents z.

$$\text{(finite-size gap)} \sim \frac{1}{L^z} \tag{1}$$

Here,

- (finite-size gap) $=$ (2nd-lowest eigval of H) $-$ (lowest eigval of H)
- L is the linear size of a system.

Typically, $z = 1$.
e.g. models described by conformal field theories, ordinary fermi liquids

There are atypical systems with $z \neq 1$.
e.g. ferromagnetic Heisenberg model, Rokhsar–Kivelson quantum dimer

e.g. XXZ chain with magnetic field

$$H = -\sum_{i=1}^{L}(X_i X_{i+1} + Y_i Y_{i+1} + \Delta Z_i Z_{i+1}) + 2h \sum_{i=1}^{L} Z_i + \text{const.} \tag{2}$$

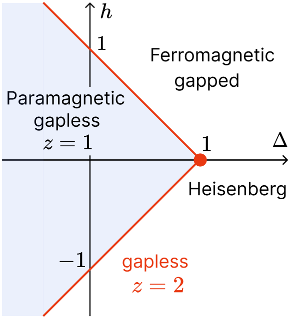

Figure 1: Phase diagram of XXZ chain with magnetic field.

When $z \neq 1$?

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Conjecture

When a gapless system is local and frustration-free, $z \geq 2$.

Definition. Frustration-free (FF) systems

A Hamiltonian H is FF if there exists a decomposition $H = \sum_i H_i$ s.t. the ground states minimize each H_i simultaneously.

Definition. Locality

Sufficiently distant local terms commute with each other.

Frustration-free systems

When a gapless Hamiltonian is local and frustration-free, then $z \geq 2$.

- Still no proof for the general claim
- There are proofs in the case of OBC.

  Gosset, Mozgunov, J. Math. Phys. 57, 091901 (2016). Anshu, PRB 101, 165104 (2020).

  Lemm, Xiang, J. Phys. A: Math. Theor. 55 295203 (2022). Lemm, Lucia, arXiv:2409.09685
- We have shown that $z \geq 2$ without assuming boundary conditions (but assuming another assumption)

Our result

- We assumed the existence of an algebraic correlation function instead of gaplessness.
- The assumption holds for a wide class of gapless FF systems.

Theorem. arXiv:2406.06415.

Let

- $D(\mathcal{O}, \mathcal{O}')$: distance between two operators
- $|\Psi\rangle$: ground state
- G: projector onto ground space

For frustration-free systems, if there exist two operators $\mathcal{O}$ and $\mathcal{O}'$ s.t.

$$D(\mathcal{O}, \mathcal{O}') \sim L \quad \text{and} \quad \frac{|\langle\Psi|\mathcal{O}(\mathbb{1}-G)\mathcal{O}'|\Psi\rangle|}{\|\mathcal{O}^\dagger|\Psi\rangle\|\|\mathcal{O}'|\Psi\rangle\|} \gtrsim \frac{1}{L^\Delta}, \tag{3}$$

then $z \geq 2$.

- The proof relies on an inequality by Gosset and Huang.

Gosset, Huang, PRL 116, 097202. (2016)

Generalized Rokhsar–Kivelson Hamiltonians of critical points

Consider generalized Rokhsar–Kivelson Hamiltonians ($\subset$ FF Hamiltonians) where GS is given by

$$|\Psi_{\mathrm{RK}}\rangle = \sum_{\mathcal{C}} \sqrt{\frac{w(\mathcal{C})}{\mathcal{Z}}}\, |\mathcal{C}\rangle, \quad \mathcal{Z} = \sum_{\mathcal{C}} w(\mathcal{C}). \tag{4}$$

- $w(\mathcal{C})$: Boltzmann weight of a classical statistical system.
- Critical statistical systems $\rightarrow$ Gapless quantum systems

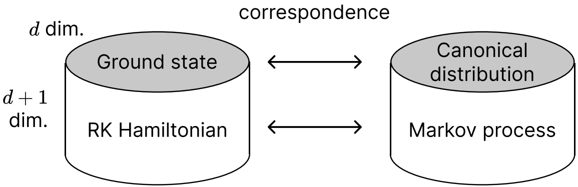
Ground State	z	References
Ising (2D)	$2.1667(5) \geq 2$	Nightingale, Blöte, PRB 62, 1089 (2000).
Ising (3D)	$2.0245(15) \geq 2$	Hasenbusch, PRE 101, 022126 (2020).
Heisenberg (3D)	$2.033(5) \geq 2$	Astillero, Ruiz-Lorenzo, PRE 100, 062117 (2019).
3-state Potts (2D)	$2.193(5) \geq 2$	Murase, Ito, JPSJ 77, 014002 (2008).
4-state Potts (2D)	$2.296(5) \geq 2$	Phys. A: Stat. Mech. Appl. 388, 4379 (2009).

Dynamic critical exponents of RK Hamiltonians of critical points

Generalized Rokhsar–Kivelson Hamiltonians of critical points

There is correspondence between RK Hamiltonians and Markov processes with the detailed balance condition.

Henley, J. Phys.: Condens. Matter 16 S891 (2004). Castelnovo *et al.*, Ann. Phys. 318, 316 (2005).



Theorem arXiv:2406.06415, arXiv:2502.09908

- RK Hamiltonians of critical points satisfy $z \geq 2$
- Local Markov processes with the detailed balance condition relaxing toward critical equilibrium states satisfy $z \geq 2$.

Proof of an empirical fact in the contexts of dynamic critical phenomena.

Summary

- Our study highlights the unique nature of the gapless FF systems.
- We have established $z \geq 2$ for a wide class of gapless FF systems.
- New fundamental result in the traditional field of dynamic critical phenomena by employing knowledge from quantum theory.

Related papers

PRB 110, 195140 (2024)

arXiv:2406.06415

arXiv:2502.09908

Related presentation

"Quadratic dispersion relations in gapless frustration-free systems"
by Haruki Watanabe
8:00 am – 8:12 am, Thursday March 20